Math 409 Midterm 1 practice #2

Name: ____________________

This exam has 3 questions, for a total of 100 points.

Please answer each question in the space provided. No aids are permitted.

Question 1. (40 pts)

In each of the following eight cases, indicate whether the given statement is true or false. No justification is necessary.

(a) Any finite subset of $\mathbb{R}$ has a least element.

(b) If E is an nonempty set such that there exists a one-to-one function $f\colon \mathbb{N} \to E$, then E is countable.

(c) If A is a nonempty subset of B, then there exists a surjective function $g\colon B \to A$.

(d) Let A be a bounded nonempty subset of $\mathbb{R}$. If $B = \{x^3 \mid x \in A\}$, then we have $\sup B = (\sup A)^3$.

(e) Let $f\colon \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + x$. Then $f([-1, 1]) = [0, 2]$.

(f) Let E be a nonempty subset of $\mathbb{R}$. Suppose E has a finite supremum and $\sup E \notin E$. Then E is an infinite set.

(g) Let A be a nonempty subset of $\mathbb{R}$. If every number in A is positive, then A has a finite infimum.

(h) There does not exist a one-to-one function from $\mathbb{R}$ to $\mathbb{N}$.

Question 2. (25 pts)

(a) State the Archimedean principle.

(b) Prove that for a given $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that

$$\frac{1}{n} < \varepsilon$$

for all $n \geq N$.

Question 3. (35 pts)

(a) State the completeness axiom for $\mathbb{R}$.

(b) Let S be a bounded nonempty subset of $\mathbb{R}$, and let a and b be fixed real numbers. Define $T = \{as + b \mid s \in S\}$. Find the formulas for $\sup T$ and $\inf T$ in terms of $\sup S$ and $\inf S$. (Just the formulas, no justification is required for this part.)

(c) Let A and B be two nonempty subsets of $\mathbb{R}$. Define

$$A + B = \{a + b \mid a \in A \text{ and } b \in B\}.$$

Prove that if both A and B are bounded above, then $\sup(A + B) = \sup A + \sup B$.